Higher Unit 17
More Algebra

Example 1

Make x the subject of the formula $P = d\sqrt{\frac{x}{y}}$

$\frac{P}{d} = \sqrt{\frac{x}{y}}$ — Divide both sides by d.

$\frac{P^2}{d^2} = \frac{x}{y}$ — Square both sides.

$\frac{yP^2}{d^2} = x$ or $x = \frac{yP^2}{d^2}$

Key point 1

When the letter to be made the subject appears twice in the formula you will need to factorise.

Example 2

Make w the subject of the formula $A = wh + lh + lw$

$A - lh = wh + lw$ — w appears twice in this formula. Subtract lh from both sides to get the terms in w together on one side of the equals sign.

$A - lh = w(h + l)$ — Factorise the right-hand side, so w appears only once.

$w = \frac{A - lh}{h + l}$ — Divide both sides by $(h + l)$.

Exam hint

First multiply both sides by $(k - 2)$.
Then expand the bracket on the left-hand side.

Exam-style question

Make k the subject of the formula $t = \frac{k}{k-2}$ **(4 marks)**

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Example 3

Simplify $\frac{x}{5} + \frac{x}{3}$

LCM of 5 and 3 is 15 — Find the LCM of the denominators.

$\frac{x}{5} = \frac{3x}{15}$ (×3) $\quad \frac{x}{3} = \frac{5x}{15}$ (×3) — Write both fractions with the same denominator.

$\frac{3x}{15} + \frac{5x}{15} = \frac{8x}{15}$ — Add the fractions.

Key point 2

You may need to factorise before simplifying an algebraic fraction:

(1) Factorise the numerator and denominator.

(2) Divide the numerator and denominator by any common factors.

Example 4

Simplify fully $\frac{x^2+5x+4}{x^2-3x-28}$

$$\frac{x^2+5x+4}{x^2-3x-28} = \frac{(x+1)(x+4)}{(x-7)(x+4)}$$

Factorise the numerator and denominator.

$$= \frac{x+1}{x-7}$$

Divide the numerator and denominator by the common factor $(x + 4)$.

Key point 3

You may need to factorise the numerator and/or denominator before you multiply or divide algebraic fractions.

Example 5

Write $\frac{7}{x+2} - \frac{3}{x+3}$ as a single fraction in its simplest form.

Common denominator $= (x+2)(x+3)$

Find a common denominator.

$$\frac{7(x+3)}{(x+2)(x+3)} - \frac{3(x+2)}{(x+2)(x+3)}$$

Convert each fraction to an equivalent fraction with the common denominator $(x+2)(x+3)$.

$$= \frac{7(x+3)-3(x+2)}{(x+2)(x+3)}$$

Subtract the fractions.

$$= \frac{7x+21-3x-6}{(x+2)(x+3)} = \frac{4x+15}{(x+2)(x+3)}$$

Expand the brackets in the numerator, then simplify.

Key point 5

A function is a rule for working out values of y for given values of x.
For example, $y = 3x$ and $y = x^2$ are functions. The notation f(x) is read as 'f of x'. f is the function.
f(x) = $3x$ means the function of x is $3x$.

Key point 6

fg is a composite function. To work out fg(x), first work out g(x) and then substitute your answer into f(x).

Exam-style question

Simplify fully

$$\frac{x^2+3x-4}{2x^2-5x+3}$$

(3 marks)

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Exam hint

1 mark is awarded for correctly factorising the numerator; 1 mark for factorising the denominator; and 1 mark for the correct final answer.

Key point 4

To rationalise the fraction $\frac{1}{a\sqrt{b}}$, multiply by $\frac{\sqrt{b}}{\sqrt{b}}$

To rationalise the fraction $\frac{1}{a \mp \sqrt{b}}$, multiply by $\frac{a \pm \sqrt{b}}{a \pm \sqrt{b}}$

Example 6

Solve $\frac{3}{2x-1} + \frac{4}{x+2} = 2$

$$\frac{3(x+2)}{(2x-1)(x+2)} + \frac{4(2x-1)}{(x+2)(2x-1)} = 2$$

Rewrite the LHS using the common denominator $(2x-1)(x+2)$.

$$\frac{3(x+2)+4(2x-1)}{(2x-1)(x+2)} = 2$$

Add the fractions.

$$\frac{3x+6+8x-4}{(2x-1)(x+2)} = \frac{11x+2}{(2x-1)(x+2)} = 2$$

Expand the brackets in the numerator and simplify.

$$11x+2 = 2(2x-1)(x+2)$$

Multiply both sides by $(2x-1)(x+2)$.

$$11x+2 = 4x^2+6x-4$$

Multiply out the brackets and simplify the right-hand side.

$$4x^2-5x-6=0$$

Rearrange into the form $ax^2+bx+c=0$.

$(4x+3)(x-2)=0$, so either $4x+3=0$ or $x-2=0$

Solve by factorising.

The solutions are $x = -\frac{3}{4}$ and $x = 2$.

Key point 7

The inverse function reverses the effect of the original function.

Example 7

Find the inverse function of $x \to 5x - 1$

Communication hint $x \to 5x - 1$ is another way of showing $f(x) = 5x - 1$

$x \to$ [×5] $\to$ [−1] $\to 5x - 1$

Write the function as a function machine.

$\frac{x+1}{5} \leftarrow$ [÷5] $\leftarrow$ [+1] $\leftarrow x$

Reverse the function machine to find the inverse function. Start with x as the input.

The inverse function of $x \to 5x - 1$ is $x \to \frac{x+1}{5}$

Key point 9

To show a statement is an identity, expand and simplify the expressions on one or both sides of the equals sign, until the two expressions are the same.

Example 8

Show that $(x + 4)^2 - 7 \equiv x^2 + 8x + 9$

LHS = $(x + 4)^2 - 7 \equiv (x + 4)(x + 4) - 7 = x^2 + 8x + 16 - 7$

$= x^2 + 8x + 9$

Expand the brackets on the left-hand side (LHS).

RHS = $x^2 + 8x + 9$

So LHS = RHS and $(x + 4)^2 - 7 \equiv x^2 + 8x + 9$

Aim to show that LHS = RHS.

Key point 8

$f^{-1}(x)$ is the inverse of $f(x)$.

strategy hint

Remember that fg means do g first and then f.

Exam hint

'Find the exact solutions' means that you should not use a calculator.

You should give your answers using simplified surds.

Even number	$2n$
Odd number	$2n + 1$ or $2n - 1$
Consecutive numbers	$n, n + 1, n + 2, \ldots$
Consecutive even numbers	$2n, 2n + 2, 2n + 4, \ldots$
Consecutive odd numbers	$2n + 1, 2n + 3, 2n + 5, \ldots$