**Higher Tier**

**Unit 1 Number**

## Key point 1

When there are $m$ ways of doing one task and $n$ ways of doing a second task, the total number of ways of doing the first task then the second task is $m \times n$.

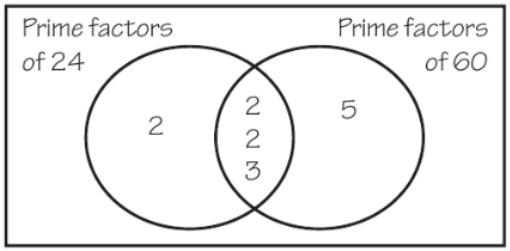
## Example 1

Find the highest common factor and lowest common multiple of 24 and 60.

$24 = 2 \times 2 \times 2 \times 3$

$60 = 2 \times 2 \times 3 \times 5$

Write each number as a product of prime factors.




Draw a Venn diagram.

The highest common factor (HCF) of 24 and 60
$= 2 \times 2 \times 3 = 12$

Multiply the common prime factors.

The lowest common multiple (LCM) of 24 and 60
$= 2 \times 2 \times 2 \times 3 \times 5 = 120$

Multiply all the prime factors.

## Key point 2

The inverse of a cube is the cube root.

$2^3 = 8$, so the cube root of 8 is $\sqrt[3]{8} = 2$

## Key point 3

To multiply, add the indices.

$x^m \times x^n = x^{m+n}$

## Key point 5

To divide powers, subtract the indices.

$x^m \div x^n = x^{m-n}$

## Key point 6

To work out a power to another power, multiply the powers together.

$(x^m)^n = x^{mn}$

## Key point 7

$x^{-n} = \frac{1}{x^n}$ for any number $n$, $x \neq 0$

## Key point 8

$x^0 = 1$, where $x$ is any non-zero number.

## Key point 9

$x^{\frac{1}{n}} = \sqrt[n]{x}$

## Key point 10

$x^{\frac{n}{m}} = (\sqrt[m]{x})^n$

## Key point 12

A number is in **standard form** when it is in the form $A \times 10^n$, where $1 \leqslant A < 10$ and $n$ is an integer.

For example, $6.3 \times 10^4$ is written is standard form because 6.3 is between 1 and 10.

$63 \times 10^4$ is *not* in standard form because 63 does not lie between 1 and 10.

Standard form is sometimes also called **scientific notation**.

## Example 3

Work out $(5 \times 10^3) \times (7 \times 10^6)$

$5 \times 7 \times 10^3 \times 10^6$ — Rewrite the multiplication grouping the numbers and the powers.

$35 \times 10^9$ — Simplify using multiplication and the index law $x^m \times x^n = x^{m+n}$. This is not in standard form because 35 is not between 1 and 10.

$35 = 3.5 \times 10^1$ — Write 35 in standard form.

$35 \times 10^9 = 3.5 \times 10^1 \times 10^9 = 3.5 \times 10^{10}$ — Work out the final answer.

## Key point 13

A **surd** is a number written exactly using square or cube roots.
For example $\sqrt{3}$ and $\sqrt[3]{5}$ are surds. $\sqrt{4}$ and $\sqrt[3]{27}$ are not surds, because $\sqrt{4} = 2$ and $\sqrt[3]{27} = 3$

## Key point 16

Rational numbers can be written as a fraction in the form $\frac{a}{b}$, where $a$ and $b$ are integers and $b \neq 0$.
2 is rational as it can be written as $\frac{2}{1}$.
$0.\dot{2}$ is rational as it can be written as $\frac{2}{9}$.
$\sqrt{2}$ is irrational.

## Example 2

Work out the value of **a** $27^{\frac{2}{3}}$ **b** $16^{-\frac{3}{4}}$

a $27^{\frac{2}{3}} = (27^{\frac{1}{3}})^2 = 3^2 = 9$ — Use the rule $(x^m)^n = x^{mn}$. Work out the cube root of 27 first. Then square your answer.

b $16^{-\frac{3}{4}} = \frac{1}{16^{\frac{3}{4}}} = \frac{1}{(16^{\frac{1}{4}})^3} = \frac{1}{2^3} = \frac{1}{8}$ — Use $x^{-n} = \frac{1}{x^n}$

## Key point 14

$\sqrt{mn} = \sqrt{m}\sqrt{n}$

## Key point 15

$\sqrt{\frac{m}{n}} = \frac{\sqrt{m}}{\sqrt{n}}$

**communication hint** In **index form** means to write a number to a power or an index.
$2^3$ is written in index form. 3 is the power or index.

**communication hint**
An integer is a positive or negative whole number or zero.

## Key point 17

To **rationalise the denominator** of $\frac{a}{\sqrt{b}}$, multiply by $\frac{\sqrt{b}}{\sqrt{b}}$. Then the fraction will have an integer as the denominator.

## Example 4

Rationalise the denominator.

a $\frac{1}{\sqrt{2}}$

b $\frac{5}{\sqrt{75}}$

a $\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{\sqrt{4}} = \frac{\sqrt{2}}{2}$ — Multiplying by $\frac{\sqrt{2}}{\sqrt{2}}$ is the same as multiplying by 1, so this does not change the value.

b $\sqrt{75} = \sqrt{25}\sqrt{3} = 5\sqrt{3}$ — First simplify $\sqrt{75}$

$\frac{5}{\sqrt{75}} = \frac{5}{5\sqrt{3}} = \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{\sqrt{9}} = \frac{\sqrt{3}}{3}$ — Simplify the fraction before rationalising.