Unit 16— Quadratic equations and graphs

Exam hint
Expand and simplify all expressions. Show your working clearly.

Example 1

Expand and simplify $(x + 2)(x + 6)$.

Grid method

$(x + 2)(x + 6)$

$\times$	x	$+2$
x	x^2	$+2x$
$+6$	$+6x$	$+12$

$= x^2 + 2x + 6x + 12$

$= x^2 + 8x + 12$

FOIL: Firsts, Outers, Inners, Lasts

$(x+2)(x+6) = x^2 + 6x + 2x + 12$

$= x^2 + 8x + 12$

Key point 2

Expanding double brackets often gives a quadratic expression.
A quadratic expression always has a squared term (with a power of 2). It cannot have a power higher than 2.
It may have a term with a power of 1 that is the same letter as the squared term.
It may also have a constant (number) term.

$x^2 + 8x + 10$

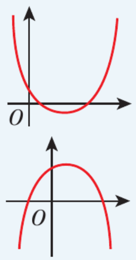
squared term — term with power 1 — constant term

Key point 3

To square a single bracket, multiply it by itself, then expand and simplify.
$(x + 1)^2 = (x + 1)(x + 1)$

Key point 5

A quadratic function has symmetrical U-shaped curve called a **parabola**.
A quadratic function with a $-x^2$ term has a symmetrical ∩-shaped curve.
The curve always has a minimum or maximum turning point.



Key point 4

The turning point of the curve is where it turns in the opposite direction.

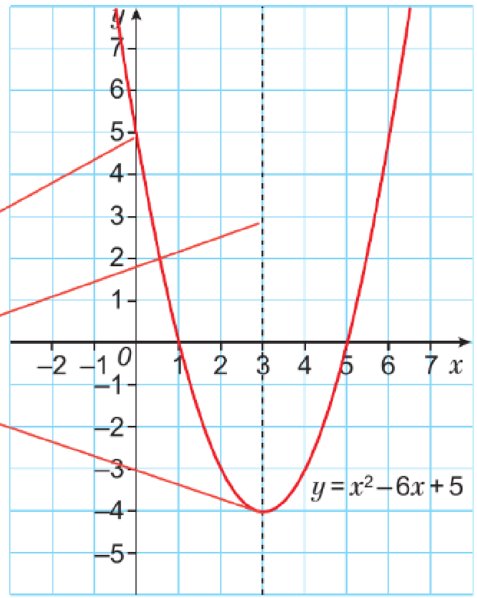
Example 2

For the graph of $y = x^2 - 6x + 5$ write down
a the equation of the line of symmetry
b the turning point
c the coordinates of the y-intercept.

The y-intercept is the point where the graph crosses the y-axis.

Sketch in the **line of symmetry**. Write its equation.

Write down the coordinates of the point where the curve turns.



a line of symmetry $x = 3$
b turning point $(3, -4)$
c y-intercept $(0, 5)$

Exam hint
Make sure the bottom of the graph is a smooth curve and not a straight line.
'Find estimates of the solutions' means read the values of the solutions from the graph as accurately as possible.

Key point 6

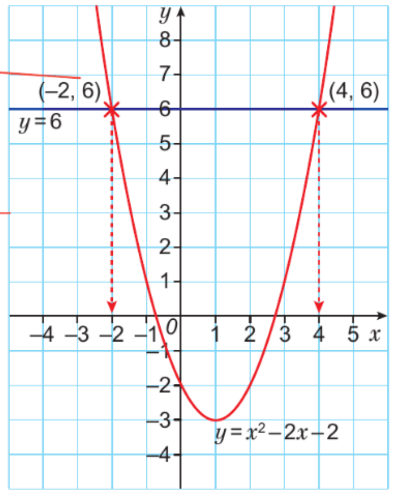
To solve the equation $ax^2 + bx + c = 0$, read the x-coordinates where the graph crosses the x-axis. The values of x that satisfy the equation are called **roots**.

Example 3

Solve the equation $x^2 - 2x - 2 = 6$

This means $y = 6$

Plot the function $y = x^2 - 2x - 2$
Draw the line $y = 6$ on the graph.



The solutions to the equation are $x = -2$ and $x = 4$

Write down the x-**coordinates** where the line and curve cross.

Example 4

Factorise $x^2 + 5x + 6$

$x^2 + 5x + 6$

Write a pair of brackets with x in each one. This gives the x^2 term when multiplied.

$(x \quad)(x \quad)$

Work out all the factor pairs of 6, the number term.

$1 \times 6 \qquad 2 \times 3$

Work out which factor pair will **add** to give 5, the number in the x term.

$1 + 6 = 7 \qquad 2 + 3 = 5$

Then write each number in each of the brackets with x

$(x + 2)(x + 3)$

Check: $(x + 2)(x + 3) = x^2 + 5x + 6$

The expression is now factorised. Expand the brackets to check it is correct.

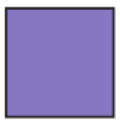
Key point 7

The **difference of two squares** is a quadratic expression with two squared terms, and one term is subtracted from the other. For example $x^2 - 25$

$x^2 \qquad 5^2$

Exam-style question

The length of a side of a square paving slab is $x + 4$.



a Work out an expression for the area of the paving slab. Simplify your answer. **(2 marks)**

b Ten paving slabs are used to make a patio. Write an expression for the total area of the patio. Simplify your answer. **(1 mark)**

Exam hint
Expand and simplify all expressions. Show your working clearly.

Key point 8

Solutions to quadratic equations can be found algebraically as well as from a graph.

Example 5

Solve $x^2 + 2x - 15 = 0$

$x^2 + 2x - 15 = 0$

$(x + 5)(x - 3) = 0$ — Factorise the quadratic expression.

$x + 5 = 0$ or $x - 3 = 0$ — As the two expressions multiply to make 0, at least one of them must equal zero.

$x = -5$ or $x = 3$ — Solve both equations.